LARGE–SCALE PRINCIPAL–AGENT PROBLEMS

In continuous–time, with volatility control

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CONTRACT THEORY

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- From a mathematical point of view:

agent behaviour $\Leftrightarrow$ stochastic control problem;

interactions $\Leftrightarrow$ Nash equilibrium and mean–field games;

incentives $\Leftrightarrow$ Stackelberg equilibrium, contract theory.

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Adverse Selection: a characteristic of the Agent is unknown by the Principal (third–best case).

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Output process: Stochastic process X with dynamic, for $t \in [0, T]$:

$$dX_t = \alpha_t dt + \sigma_t dW_t.$$

Effort: the Agent controls X through the drift α, in order to maximise the following criteria:

$$\mathbb{E}^{\mathbb{P}^\alpha}\bigg[-\exp\bigg(-R_A\Big(\xi - \int_0^T c(\alpha_t)dt\Big)\bigg)\bigg].$$

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- The **optimal** form of contracts for the Agent is (see [11]):

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where

(i) Z is a payment rate chosen by the Principal;
(ii) $\mathcal{H}$ is the Agent's Hamiltonian.

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(i) identify a class of contracts, offered by the principal, that are revealing: the agent's optimal response can be easily calculated;

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- Use these recent developments to:

  (i) identify the optimal incentives within a hierarchy;

  (ii) improve electricity demand management.

INCENTIVES WITHIN A HIERARCHY

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- Answer two questions:

(i) Interest of continuous–time?

(ii) 'Natural' example where an agent controls the volatility?

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The agents are the $n+1$ workers, with CARA utility. Each agent $i \in \{0, \dots, N\}$ produce a random outcome X^i, by carrying out his own task:

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$$X^i = \alpha^i + \sigma^i W^i, \quad W^i \sim \mathcal{N}(0,1) \text{ i.i.d.}$$

The effort of the agent i is represented by α^i, and induces a quadratic cost: $c^i(\alpha^i) = |\alpha^i|^2/2k^i$.

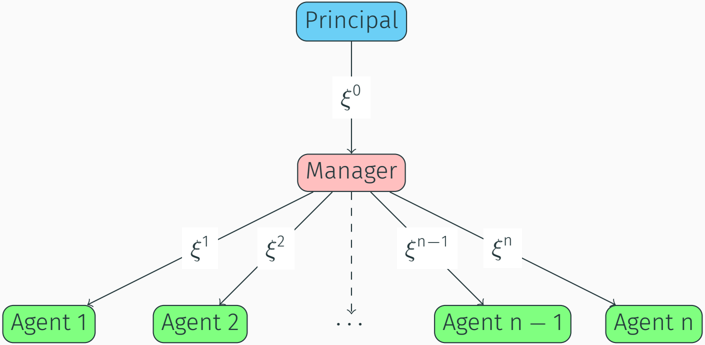

Figure: Sung's model

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(iii) The principal observes only the net profit of the hierarchy :

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- Interconnected principal–agent problems.

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where Z^i is a parameter chosen by the manager.

- The agent's optimal effort is the one which maximise his Hamiltonian: $k^i Z^i$.

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- The variance of ζ is not observed by the principal, and therefore the **contract for the manager** cannot be indexed on it:

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The i–th agent

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The principal observes in continuous–time the process ζ :

$$\zeta_t = \sum_{i=0}^{n} X^i_t - \sum_{i=1}^{n} \xi^i_t,\ t \in [0,1],$$

and indexes the contract ξ^0 for the manager on this process.

- Given a contract ξ^i, the i–th agent chooses an effort α^i in order to maximise the following utility:

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▶ The optimal effort of the agent is $k^iZ_t^i$, and it is possible to compute the dynamics of X^i and ξ^i under this optimal effort.

- Given a contract ξ^0, the manager chooses α^0 and Z^i, for $i = 1, \ldots, n$, in order to maximise:

$$\mathbb{E}^{\mathbb{P}^0}\left[-\exp\left(-R^0\left(\xi^0 - \int_0^1 c^0(\alpha_t^0)\mathrm{d}t\right)\right)\right].$$

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Hypothesis: the principal observes only ζ, which satisfies:

$$\mathrm{d}\zeta_t = \mathrm{d}X_t^0 + \sum_{i=1}^n \left(k^i Z_t^i - \frac{1}{2}(Z_t^i)^2\left(k^i + R^i(\sigma^i)^2\right)\right)\mathrm{d}t + \sigma^i \sum_{i=1}^n (1 - Z_t^i)\mathrm{d}W_t^i.$$

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$$d\zeta_t = dX_t^0 + \sum_{i=1}^n \Big(k^i Z_t^i - \frac{1}{2}(Z_t^i)^2\big(k^i + R^i(\sigma^i)^2\big)\Big)dt + \sigma^i \sum_{i=1}^n (1 - Z_t^i)dW_t^i.$$

- The manager controls the volatility of his state variable ζ.
- The optimal of contract is (Cvitanić, Possamaï, and Touzi [4] (2018)):

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- Given a contract ξ^0, the manager chooses α^0 and Z^i, for $i = 1, \dots, n$, in order to maximise:

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- Explicit forms for the manager's optimal controls and dynamics for ζ and ξ^0 at the optimum.

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    - Difference with the one–period model lies: in continuous time, the principal observes the quadratic variation of ζ.
    - Shows the need to rigorously study continuous–time, and therefore to use second–order BSDEs.

- This model can be extended to a more general framework, in terms of:

(i) hierarchy;
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- What would happen in a mean–field framework?

TOWARDS A MEAN–FIELD OF AGENTS

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- Problem : Inflexible (or expensive) production and random renewable energies.
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- Contribution, with Elie, Mastrolia and Possamaï: extension of [1] to a model with a continuum of agents, whose electricity consumption is impacted by a common noise, representing climatic hazards.

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$$V_0^A(\xi) := \sup_{\nu=(\alpha,\beta)} \mathbb{E}^{\mathbb{P}}\left[U_A\left(\xi - \int_0^T \left(c(\nu_t) - f(X_t)\right)\mathrm{d}t\right)\right], \tag{5}$$

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- Principal – multi–agents models : the principal can take advantage of the supplementary information available to her (see [5, 6]).

- In our case, the Principal can compute the distribution, conditional to common noise, of the deviation of the other consumers, denoted $\hat{\mu}$.

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  - $\widehat{\alpha}^\star$, the optimal effort of the others on the drift of their deviation,
  - $\widehat{X}$, the deviation of others;
  - $\widehat{\mathbb{E}}^{\hat{\mu}}$, expectation under $\hat{\mu}$ (with respect to the common noise).

Equilibrium between agents: Given a contract of the previous form, indexed by (Z, Z^μ, Γ),

- the optimal effort of the representative agent is the same as in [1] and does not depend on Z^μ or $\hat{\mu}$;

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- from the Principal's point of view, the contract ξ is a function of X and μ^X, the conditional law of X. $\Leftrightarrow$ Problem of McKean–Vlasov type.

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- Risk–neutral case ($R_P = 0$) $\Rightarrow$ Classic contract for drift and volatility control, indexed on X°, the part of the deviation that is actually controlled by the agent.

CONCLUSION

Theoretical contribution: Extension of PA problems with volatility control to a multitude / continuum of agents, by developing natural extensions of the 2BSDE theory.

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Applications:

- modelling of interactions and incentives in an organisation;
- demand–response management;
- control of an epidemic (see Aurell, Carmona, Dayanikli, and Lauriere [2] (2020));
- finance, insurance...

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